HOMEWORK 2

YOUR NAME

Due date: Oct. 9

Exercise: 2, 3, 5, 8, page 10-11 of Hoffman-Kunze.

Problem 1. *Let F be a field with positive characteristic. Show that* $\mathrm{char}(F)$ *is prime.*

(Recall that F has finite characteristic means there exists a positive integer n such that $n \cdot 1 = 1 + 1 + \cdots + 1 = 0$). $\mathrm{char}(F) = \min \{n | n \cdot 1 = 0\}$.

Problem 2. *Suppose that F is a field of characteristic zero. Show that there exists an embedding (namely, an injective map) $f : \mathbb{Q} \to F$ such that*

$$f(a+b) = f(a) + f(b);$$
$$f(ab) = f(a) \cdot f(b),$$

for all $a, b \in F$.

Here keep in mind that in $a + b$, the sign $+$ means the addition in $\mathbb{Q}$, while the plus sign $+$ in $f(a) + f(b)$ means the addition in F. Similarly, the product notations in the second line have different meanings. This is Exercise 8, page 5. Similarly, try to show the following: let F be a field of characteristic $p > 0$, (here p must be a prime number by Problem 1), then there exists an injective map $f : \mathbb{F}_p \to F$ which satisfying $f(a + b) = f(a) + f(b), f(ab) = f(a) \cdot f(b)$ for any $a, b \in \mathbb{F}_p$. You don't have to write up the details in your homework but try to work this out.

Additional Exercise: Exercise 6 of page 5, Exercise 7 of page 11. Think about these two problems at least in some special cases (for example, in Exercise 6, page 5, you can assume that each linear system has only two unknowns and two equations; in exercise 7, page 11, you can assume that the matrix is 2×2). But there is no need to submit it. We will come back to these problems in the future.