HOMEWORK 10

Due date: Monday of Week 11, May 6,

Exercises: 4.9, 4.10, 4.11, 4.12, 4.13, 4.14,
The Exercises are from the document "Notes on tensor and exterior products".

Problem 1. *Let S_1, S_2 be two sets and let $f : S_1 \to S_2$ be a map between sets. For $i = 1, 2$, let F^{S_i} be the free vector space over S_i with the structure map $\iota_{S_i} : S_i \to F^{S_i}$. Show that there is a unique linear map $\phi : F^{S_1} \to F^{S_2}$ such that*

$$\phi \circ \iota_{S_1} = \iota_{S_2} \circ f.$$

Moreover, if f is bijective, show that ϕ is also bijective.

Problem 2. *Let V_1, V_2 be two vector spaces, show that the tensor product $(V_1 \otimes_F V_2, \tau)$ as defined in Definition 4.3 is unique up to a unique isomorphism.*

This is Proposition 4.4.