HOMEWORK 11

Due date: Monday of Week 12

Exercises, 5.3, 6.9, 6.10, 7.3, 7.4, 7.7, 7.8, 7.11 from the document "Notes on tensor and exterior products".

The following are exercises from M. Artin's book "Algebra", 2nd edition.

Exercises: 2.4, 2.6, 3.1, 3.2, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.8,9.4, 9.6, pages 69-75

For Exercises in Section 9, just remind you the notations. $2x \equiv 5$ modulo 9 just means that $2x = 5$ in $\mathbb{Z}/9\mathbb{Z}$. For the Chinese Remainder Theorem of Ex.9.6, recall that we have proved the following. Let F be a field, and $f, g \in F[x]$ be relative prime polynomials, then

$$F[x]/(fg) \cong F[x]/(f) \times F[x]/(g).$$

Try to interpret Ex.9.6 using a similar isomorphism as above. You can also try to prove that isomorphism.

Problem 1. *Let K be a field and $F \subset K$ is a subfield. Let V, W be two vector spaces over F and let $T : V \to W$ be an F-linear map. Consider the T-linear map $T \otimes \mathrm{id} : V \otimes_F K \to W \otimes_F K$. Show that*

$$\mathrm{Ker}(T \otimes \mathrm{id}) = \mathrm{Ker}(T) \otimes_F K.$$

Problem 2. *Let K be a field and $F \subset K$ is a subfield. Let V be a vector space over F. Let $S = \{\alpha_1, \ldots, \alpha_n\} \subset V$ be a subset of V and consider $S' = \{\alpha_1 \otimes 1, \ldots, \alpha_n \otimes 1\} \subset V \otimes_F K$. Show that S is linearly independent over F if and only if S' is linearly independent over K.*