

## HOMWORK 10

Due date: Next Monday,

Exercises: 4.9, 4.10, 4.11, 4.12, 4.13, 4.14, 5.3, 6.9, 6.10, 6.19, 6.24.

The Exercises are from the document “[Notes on multi-linear algebra](#)”.

**Problem 1.** Let  $S_1, S_2$  be two sets and let  $f : S_1 \rightarrow S_2$  be a map between sets. For  $i = 1, 2$ , let  $F^{S_i}$  be the free vector space over  $S_i$  with the structure map  $\iota_{S_i} : S_i \rightarrow F^{S_i}$ . Show that there is a unique linear map  $\phi : F^{S_1} \rightarrow F^{S_2}$  such that

$$\phi \circ \iota_{S_1} = \iota_{S_2} \circ f.$$

Moreover, if  $f$  is bijective, show that  $\phi$  is also bijective.

**Problem 2.** Let  $V_1, V_2$  be two vector spaces, show that the tensor product  $(V_1 \otimes_F V_2, \tau)$  as defined in Definition 4.3 is unique up to a unique isomorphism.

This is Proposition 4.4.