

HOMEWORK 11

Due date: Monday of Week 12

The following are exercises from M. Artin's book "Algebra", 2nd edition.

Exercises: 2.4, 2.6, 3.1, 3.2, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.8, 5.1, 5.2, 5.3, 5.6, 6.2, 6.3, 6.5, 6.6, 6.7, 6.10, 6.11, 9.4, 9.6, pages 69-75

For Exercises in Section 9, just remind you the notations. $2x \equiv 5$ modulo 9 just means that $2x = 5$ in $\mathbb{Z}/9\mathbb{Z}$. For the Chinese Remainder Theorem of Ex.9.6, recall that we have proved the following. Let F be a field, and $f, g \in F[x]$ be relative prime polynomials, then

$$F[x]/(fg) \cong F[x]/(f) \times F[x]/(g).$$

Try to interpret Ex.9.6 using a similar isomorphism as above. You can also try to prove that isomorphism.

Problem 1. Let K be a field and $F \subset K$ is a subfield. Let V, W be two vector spaces over F and let $T : V \rightarrow W$ be an F -linear map. Consider the T -linear map $T \otimes \text{id} : V \otimes_F K \rightarrow W \otimes_F K$. Show that

$$\text{Ker}(T \otimes \text{id}) = \text{Ker}(T) \otimes_F K.$$

Problem 2. Let K be a field and $F \subset K$ is a subfield. Let V be a vector space over F . Let $S = \{\alpha_1, \dots, \alpha_n\} \subset V$ be a subset of V and consider $S' = \{\alpha_1 \otimes 1, \dots, \alpha_n \otimes 1\} \subset V \otimes_F K$. Show that S is linearly independent over F if and only if S' is linearly independent over K .